ON THE MOTION OF AN ASTATIC GYROSCOPE IN A CARDAN SUSPENSION WITH DRY FRICTION

(O DVIZHENII ASTATICHESKOGO GIROSKOPA V KARDANOVOM PODVESE S SUKHIM TRENIEM)

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D.M. KLIMOV
(Moscow)

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Nikolai [1,2] formulated and solved the problem of the motion of a stabilized gyroscope in a Cardan suspension taking into account the effect of constant frictional moments at the suspension axes. The motion of an astatic gyroscope in a Cardan suspension on a fixed base is studied below. It is assumed that there are frictional forces at the axes of the suspension, the magnitudes of which are proportional to the normals forming the dynamic reactions. Some results have been published earlier [3].

1. Derivation of the equations of motion for the gyroscope. Let us associate the system of coordinates $\xi\eta\zeta$ with the space at rest, the system of coordinates x_1, y_1, z_1 with the outer ring, and x_2, y_2, z_2 with the inner ring; furthermore, the axis of rotation for the outer ring coincides with the axis ξ, the axis y_1 with the axis of rotation of the inner ring, the axis z_2 with the rotational axis of the rotor. The location of the gyroscopic system will be denoted by the angles α, β and ϕ, the sense of which is shown in Fig. 1.

Let us denote the moment of inertia of the outer ring about the axis of its rotation by A_1, the moments of inertia of the inner rings about the axes $x_2 y_2 z_2$ by $A_2 B_2 C_2$, the equatorial and polar moments of inertia of the rotor by A and C. Denoting the projection of the angular velocity of the outer ring on the axes x_2, y_2, z_2 by p_2, q_2, r_2, the projection of the angular velocity of the rotor on the same axes by p, q, r, we have

$$\begin{gathered}
p_2 = \alpha' \cos\beta, \qquad p = \alpha' \cos\beta \\
p_1 = \alpha', \qquad q_2 = \beta', \qquad q = \beta' \\
r_2 = \alpha' \sin\beta, \quad r = \varphi' + \alpha' \sin\beta \\
\left(\alpha' = \frac{d\alpha}{dt},\ \beta' = \frac{d\beta}{dt},\ \varphi' = \frac{d\varphi}{dt}\right)
\end{gathered} \tag{1.1}$$

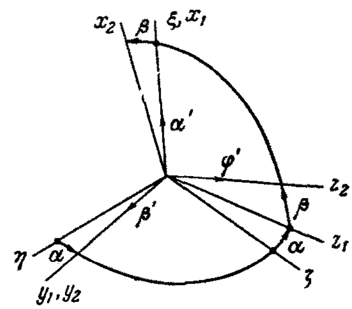

Fig. 1.

Denoting, finally, by K_{x1}, K_{y1}, K_{z1} the sums of the moments of forces of base reactions with respect to the axes x_1, y_1, z_1, by L_{x1}, L_{y1}, L_{z1} the sums of the moments of forces acting on the inner ring from the side of the outer ring, by M_{x2}, M_{y2}, M_{z2} the moments exerted by the inner ring on the rotor [4]:

Then, utilizing (1.1) we will obtain the system of equations describing the motion of the outer ring, the inner ring and the rotor:

$$
\begin{gathered}
A_1\alpha'' = K_{x_1} - L_{x_1}, \qquad 0 = K_{y_1} - L_{y_1}, \qquad 0 = K_{z_1} - L_{z_1} \\
A_2\alpha'' \cos\beta - (A_2 + B_2 - C_2)\,\alpha'\beta' \sin\beta = L_{x_1}\cos\beta - L_{z_1}\sin\beta - M_{x_2} \\
B_2\beta'' + (A_2 - C_2)\,\alpha'^2 \sin\beta\cos\beta = L_{y_1} - M_{y_2} \qquad (1.2) \\
C_2\alpha'' \sin\beta + (C_2 + B_2 - A_2)\,\alpha'\beta' \cos\beta = L_{x_1}\sin\beta + L_{z_1}\cos\beta - M_{z_2} \\
A\alpha'' \cos\beta + H\beta' - 2A\alpha'\beta' \sin\beta = M_{x_2} \\
A\beta'' + A\alpha'^2 \sin\beta\cos\beta - H\alpha' \cos\beta = M_{y_2} \\
\frac{d}{dt}\left[C\,(\varphi' + \alpha' \sin\beta)\right] = \frac{dH}{dt} = M_{z_2}
\end{gathered}
$$

Let us consider the forces of interaction between the base and the outer ring. Let us assume that the journal and the bearing in cross-section, perpendicular to the axis of rotation, represent two circles, the radii of which differ insignificantly. Figure 2 shows the cross-section of the outer ring bearing located on the positive part of the axis x_1. The normal force of the base reaction on the outer ring is denoted by R_1, where $R_1 > 0$; the friction force $F_1 = f_1R_1$ is directed perpendicularly toward R_1 and hinders the rotation of the outer ring. The forces in the second bearing have the same magnitudes but opposite directions.

We will assume that the axis of outer ring rotation is directed vertically; then, denoting by r_1 the radius of the bearing, by l_1 the length of outer ring axis, we have

$$K_{x_1} = -(2f_1r_1R_1 + K_{x_1}^*)\,\mathrm{sign}\,\alpha'$$
$$K_{y_1} = -R_1l_1(\sin\vartheta_1 + f_1\cos\vartheta_1\,\mathrm{sign}\,\alpha') \qquad (1.3)$$
$$K_{z_1} = R_1l_1(\cos\vartheta_1 - f_1\sin\vartheta_1\,\mathrm{sign}\,\alpha')$$

where K_{x1}^* is the moment of rotational friction dependent on the force of gravity. Because the axis of the outer ring is directed vertically, gravity has no effect on R_1.

Let us pass now to the consideration of reaction forces of the outer ring on the inner ring. The cross-sections of the bearings located on the positive and negative sides of the axis y_1 are shown in Figs. 3 and 4. Utilizing the given notationswe will get

$$L_{x_1} = R_{21}(\cos\vartheta_{21} - f_{21}\sin\vartheta_{21}\,\mathrm{sign}\,\beta')\frac{l_2}{2} + R_{22}(\cos\vartheta_{22} + f_2\sin\vartheta_{22}\,\mathrm{sign}\,\beta')\frac{l_2}{2}$$
$$L_{y_1} = -f_2r_2(R_{21} + R_{22})\,\mathrm{sign}\,\beta' \qquad (1.4)$$
$$L_{z_1} = -R_{21}(\sin\vartheta_{21} + f_{21}\cos\vartheta_{21}\,\mathrm{sign}\,\beta')\frac{l_2}{2} + R_{22}(\sin\vartheta_{22} - f_2\cos\vartheta_{22}\,\mathrm{sign}\,\beta')\frac{l_2}{2}$$

Here r_2 is the radius of the bearing and l_2 the length of the inner ring axis.

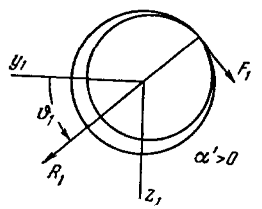

Fig. 2.

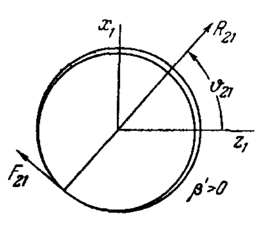

Fig. 3.

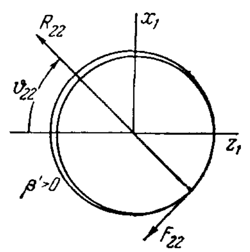

Fig. 4.

Denoting the weight of the rotor and of the inner ring by P and projecting the forces acting on the inner ring on the axes x_1 and z_1, we have

$$R_{21}(\sin\vartheta_{21} + f_2\cos\vartheta_{21}\,\mathrm{sign}\,\beta') + R_{22}(\sin\vartheta_{22} - f_2\cos\vartheta_{22}\,\mathrm{sign}\,\beta') = P$$
$$R_{21}(\cos\vartheta_{21} - f_2\sin\vartheta_{21}\,\mathrm{sign}\,\beta') - R_{22}(\cos\vartheta_{22} + f_2\sin\vartheta_{22}\,\mathrm{sign}\,\beta') = 0 \qquad (1.5)$$

Using (1.2) to (1.5), we obtain a system of equations describing the motion of the gyroscope in a Cardan suspension with dry friction:

$$A_1\alpha'' = -(2f_1r_1 + K_{x_1}^*)\,\mathrm{sign}\,\alpha' - R_{21}(\cos\vartheta_{21} - f_2\sin\vartheta_{21}\,\mathrm{sign}\,\beta')\,l_2 \qquad (1.6)$$
$$0 = -R_1l_1(\sin\vartheta_1 + f_1\cos\vartheta_1\,\mathrm{sign}\,\alpha') + f_2r_2(R_{21} + R_{22})\,\mathrm{sign}\,\beta' \qquad (1.7)$$

$$0 = R_1 l_1 (\cos\vartheta_1 - f_1 \sin\vartheta_1 \operatorname{sign}\alpha') + R_{21} (\sin\vartheta_{21} + f_2 \cos\vartheta_{21} \operatorname{sign}\beta') l_2 - {}^1/_2 P l_2 \quad (1.8)$$

$$A_2\alpha'' \cos\beta - (A_2 + B_2 - C_2)\alpha'\beta' \sin\beta = R_{21}(\cos\vartheta_{21} - f_2 \sin\vartheta_{21} \operatorname{sign}\beta') l_2 \cos\beta +$$
$$+ [R_{21}(\sin\vartheta_{21} + f_2 \cos\vartheta_{21} \operatorname{sign}\beta') - {}^1/_2 P] l_2 \sin\beta - M_{x_2} \quad (1.9)$$

$$B_2\beta'' + (A_2 - C_2)\alpha'^2 \sin\beta\cos\beta = -f_2 r_2 (R_{21} + R_{22}) \operatorname{sign}\beta' - M_{y_2} \quad (1.10)$$

$$C_2\alpha'' \sin\beta + (C_2 + B_2 - A_2)\alpha'\beta' \cos\beta = R_{21}(\cos\vartheta_{21} - f_2 \sin\vartheta_{21} \operatorname{sign}\beta') l_2 \sin\beta -$$
$$- [R_{21}(\sin\vartheta_{21} + f_2 \cos\vartheta_{21} \operatorname{sign}\beta') - {}^1/_2 P] l_2 \cos\beta - M_{z_2} \quad (1.11)$$

$$A\alpha'' \cos\beta + H\beta' - 2A\alpha'\beta' \sin\beta = M_{x_2} \quad (1.12)$$

$$A\beta'' + A\alpha'^2 \sin\beta\cos\beta - H\alpha' \cos\beta = M_{y_2} \quad (1.13)$$

$$\frac{dH}{dt} = M_{z_2} \quad (1.14)$$

These sould be supplemented with the relations (1.5).

2. Motion of the gyroscope under the action of a constant moment. We will consider the motion of a gyroscope under the action of a constant moment outside the gravity field (the case of motion in the absence of external influences is presented in [3]).

Let us assume that there is no friction on the rotor axis, i.e. $M_{z2} = 0$, and the constant moment M is applied to the outer ring along its axis. From (1.14) we find that $H = \text{const}$.

Since $P = 0$, then from (1.5) it can be seen that $R_{21} = R_{22}$.

We will assume that the angles α, β and the angular velocities α', β' are small quantities and will neglect their squares, products and terms of the order $\alpha''\beta$. Then, for example,

$$M_{x_2} = A\alpha'' + H\beta', \quad M_{x_2} \sin\beta \approx (A\alpha'' + H\beta')\beta \approx 0.$$

Using (1.9) to (1.13) we obtain

$$R_{21}(\cos\vartheta_{21} - f_2 \sin\vartheta_{21} \operatorname{sign}\beta') l_2 = (A + A_2)\alpha'' + H\beta'$$
$$-2f_2 r_2 R_{21} \operatorname{sign}\beta' = (A + B_2)\beta'' - H\alpha'$$
$$R_{21}(\sin\vartheta_{21} + f_2 \cos\vartheta_{21} \operatorname{sign}\beta') l_2 = 0$$

Squaring the first and the third equation of the last system and adding them we have

$$R_{21} = \frac{1}{l_2\sqrt{1 + f_2^2}} |(A + A_2)\alpha'' + H\beta'|$$

After eliminating R_{21} we find

$$(A + B_2)\beta'' - H\alpha' = -\frac{2f_2 r_2}{l_2\sqrt{1 + f_2^2}} |(A + A_2)\alpha'' + H\beta'| \operatorname{sign}\beta' \quad (2.1)$$

Eliminating R_1 and ϑ_1 from Equations (1.6) to (1.8), we obtain

$$(A + A_1 + A_2)\,\alpha'' + H\beta' = -\frac{2f_1r_1}{l_1\sqrt{1+f_1^2}}\,|\,(A+B_2)\,\beta'' - H\alpha'\,|\,\mathrm{sign}\,\alpha' + M \qquad (2.2)$$

Let us introduce

$$\alpha' = x, \qquad \beta' = y, \qquad A + A_1 + A_2 = I_1, \qquad A + A_2 = I_2, \qquad A + B_2 = I_3$$
$$\frac{2f_1r_1}{l_1\sqrt{1+f_1^2}} = a_1 > 0, \qquad \frac{2f_2r_2}{l_2\sqrt{1+f_2^2}} = a_2 > 0$$

Equations (2.1) and (2.2) can be rewritten in the following form:

$$\begin{aligned} I_1x' + Hy &= -a_1\,|\,I_3y' - Hx\,|\,\mathrm{sign}\,x + M \\ I_3y' - Hx &= -a_2\,|\,I_2x' + Hy\,|\,\mathrm{sign}\,y \end{aligned} \qquad (2.3)$$

The first equation of the system (2.3) can be put into the form

$$I_1x' + Hy = -a_1a_2\,|\,I_2x' + Hy\,|\,\mathrm{sign}\,x + M \qquad (2.4)$$

Whence

$$x' = \frac{M - (1 \pm a_1a_2\,\mathrm{sign}\,x)\,Hy}{I_1 \pm a_1a_2I_2\,\mathrm{sign}\,x} \qquad \begin{matrix}\text{plus for } I_2x' + Hy > 0 \\ \text{minus for } I_2x' + Hy < 0\end{matrix} \qquad (2.5)$$

Taking into account (2.5) we obtain

$$I_2x' + Hy = \frac{MI_2 + A_1Hy}{I_1 \pm a_1a_2I_2\mathrm{sign}\,x}, \quad \text{or} \quad \begin{cases} I_2x' + Hy > 0 & \text{for } y > y_0 \\ I_2x' + Hy = 0 & \text{for } y = y_0 \\ I_2x' + Hy < 0 & \text{for } y < y_0 \end{cases} \quad \left(y_0 = -\frac{I_2}{A_1}\frac{M}{H}\right)$$

Let us consider the motion of a representative point $Q(x, y)$ on the plane of the angular velocities y, x, introduced by Nikolai.

Equation (2.3) shows that the plane of Nikolai can be split into six regions in each of which the motion of the gyroscope is described by linear equations. In each region these equations can be reduced by transformation of coordinates to the equation of the form

$$\frac{dy}{dx} = -m_i\frac{x}{y} \pm n_i \qquad (i = 1,2) \qquad (2.6)$$

in which

$$m_1 = \frac{I_1 + a_1a_2I_2}{I_3(1 + a_1a_2)}, \qquad m_2 = \frac{I_1 - a_1a_2I_2}{I_3(1 - a_1a_2)}, \qquad n_1 = \frac{a_2A_1}{I_3(1 + a_1a_2)}, \qquad n_2 = \frac{a_2A_1}{I_3(1 - a_1a_2)}$$

From Equation (2.6) it follows that the motion of the representative point occurs along deformed logarithmic spirals (Fig. 5), the centers of which correspond to the six regions of the Nikolai plane.

	Region	Spiral Centers
I	$(x>0,\ y>0)$	$O_1\left(\frac{a_2}{1+a_1a_2}\frac{M}{H},\ \frac{1}{1+a_1a_2}\frac{M}{H}\right)$
II	$(x<0,\ y>0)$	$O_2\left(\frac{a_2}{1-a_1a_2}\frac{M}{H}\ \ \frac{1}{1-a_1a_2}\frac{M}{H}\right)$
III	$(x<0,\ y_0<y<0)$	$O_3\left(-\frac{a_2}{1-a_1a_2}\frac{M}{H},\ \frac{1}{1-a_1a_2}\frac{M}{H}\right)$
IV	$(x<0,\ y<y_0)$	$O_4\left(\frac{a_2}{1+a_1a_2}\frac{M}{H},\ \frac{1}{1+a_1a_2}\frac{M}{H}\right)$
V	$(x>0,\ y<y_0)$	$O_5\left(\frac{a_2}{1-a_1a_2}\frac{M}{H},\ \frac{1}{1-a_1a_2}\frac{M}{H}\right)$
VI	$(x>0,\ y_0<y<0)$	$O_6\left(-\frac{a_2}{1+a_1a_2}\frac{M}{H},\ \frac{1}{1+a_1a_2}\frac{M}{H}\right)$

Let us consider the passing of the representative point through the coordinate axes. If the representative point crosses the y-axis then the angular velocity of the outer ring $\alpha' = x$ vanishes and in accordance with Equation (2.4) there may be two cases:

$$(1) \qquad |M - Hy| > a_1a_2\,|I_2x' + Hy|, \qquad x' \neq 0,$$

the representative point passes across the y-axis;

$$(2) \qquad |M - Hy| \leqslant a_1a_2\,|Hy|, \qquad x' = 0$$

the representative point slides along the y-axis. The slide region is defined by the inequality

$$y_1 \leqslant y \leqslant y_2 \qquad \left(y_1 = \frac{1}{1+a_1a_2}\frac{M}{H},\ y_2 = \frac{1}{1-a_1a_2}\frac{M}{H}\right)$$

The pattern of motion of the representative point verifies the existence of the slide region on the y-axis. Indeed, from (2.4) it is seen that if $x > 0$, $y = y_1$, then $x' = 0$; if $x < 0$, $y = y_2$, then also $x' = 0$. The straight lines $x' = 0$ are shown in Fig. 6 by dotted lines. Above the dotted lines the motion of the representative point occurs from left to right. The region y_1y_2 attracts the representative point by the only possible motion which is sliding along the y-axis towards the origin of the coordinates.

Analogous reasoning shows that the slide region exists on the x-axis (Fig. 6) and is defined by the inequality

$$x_1 \leqslant x \leqslant x_2 \quad \left(x_1 = -a_2\frac{I_2M}{(I_1 - a_1a_2I_2)H},\ x_2 = a_2\frac{I_2M}{(I_1 + a_1a_2I_2)H}\right)$$

At the point x_2 the representative point leaves the x-axis, passes into the region *I* and with increasing time approaches O_1 asymptotically. Thus, after the transient process dies down in the system, two constant angular

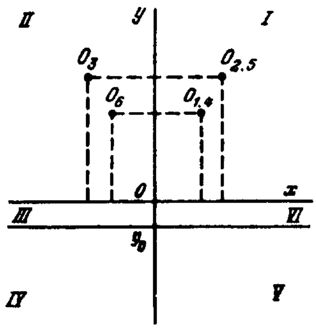

Fig. 5.

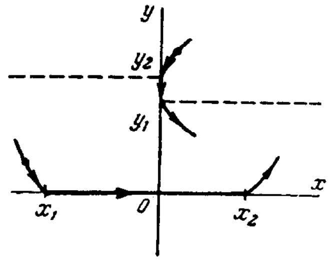

Fig. 6.

velocities are established:

$$\beta' = \frac{1}{1 + a_1 a_2} \frac{M}{H}, \qquad \alpha' = \frac{a_2}{1 + a_1 a_2} \frac{M}{H}$$

Let us assume now that an external constant moment L is applied to the inner ring along its axis of rotation. A simple analysis shows that in the system, subsequent to the transient process, there is established one constant angular velocity $\alpha' = L/H$. In this case the slide regions of the representative point along the coordinate axes are non-existent.

3. On the motion of a heavy gyroscope. Let us assume again that $M_{z2} = 0$, $H =$ const and consider the motion of a heavy gyroscope, retricting ourselves to the case of small angles and small velocities. From (1.11) and (1.5) it can be seen that

$$R_{21} (\sin \vartheta_{21} + f_2 \cos \vartheta_{21} \operatorname{sign} \beta') = R_{22} (\sin \vartheta_{22} - f_2 \cos \vartheta_{22} \operatorname{sign} \beta') = \frac{P}{2}, \quad \text{or} \quad R_{21} = R_{22}$$

Using Equations (1.9) to (1.13) and the last relationship we find

$$L_{x_1} = R_{21} (\cos \vartheta_{21} - f_2 \sin \vartheta_{21} \operatorname{sign} \beta') \, l_2 = (A + A_2) \alpha'' + H\beta'$$

$$L_{y_1} = -2 f_2 r_2 R_{21} \operatorname{sign} \beta' = (A + B_2) \beta'' - H\alpha'$$

$$L_{z_1} = \left[\frac{P}{2} - R_{21} (\sin \vartheta_{21} + f_2 \cos \vartheta_{21} \operatorname{sign} \beta') \right] l_2 = 0$$

Let us introduce the notation $1/2\, P l_2 = m$. Then eliminating R_{21} and ϑ_{21}, we have

$$(A + B_2) \beta'' - H\alpha' = -a_2 \sqrt{m^2 + [(A + A_2) \alpha'' + H\beta']^2} \operatorname{sign} \beta' \qquad (3.1)$$

We will assume that the bearings of the outer ring are designed in such a way that the moment $K_{x1}{}^*$ due to rotating friction dependent on gravity can be neglected.

After eliminating R_1 and ϑ_1 from Equations (1.6) to (1.8), we obtain

$$(A + A_1 + A_2)\,\alpha'' + H\beta' = -a_1 \,|\, (A + B_2)\,\beta'' - H\alpha' \,|\, \operatorname{sign}\alpha' \qquad (3.2$$

Utilizing the notation from the previous section, we write Equations (3.1) and (3.2) in the following form:

$$\begin{aligned} I_1 x' + Hy &= -a_1 a_2 \sqrt{m^2 + (I_2 x' + Hy)^2}\operatorname{sign} x \\ I_3 y' - Hx &= -a_2 \sqrt{m^2 + (I_2 x' + Hy)^2}\operatorname{sign} y \end{aligned} \qquad (3.3)$$

First consider the case when, due to smallness of friction, one can neglect the right-hand side of the first equation in system (3.3). Then

$$x' = -\frac{H}{I_1} y, \qquad I_3 y' - Hx = -a_2 \sqrt{m^2 + n^2 H^2 y^2}\operatorname{sign} y \qquad \left(n = \frac{A_1}{I_1}\right)$$

whence

$$\frac{dy}{dx} = -\frac{I_1}{I_3}\,\frac{Hx - a_2 \sqrt{m^2 + n^2 H^2 y^2}\operatorname{sign} y}{Hy}$$

Integrating this equation we will obtain

$$\begin{aligned} &\ln\left(m_1^2 + n^2 y^2 - a_2 bx \sqrt{m_1^2 + n^2 y^2}\operatorname{sign} y + bx^2\right) = \\ &= D - \frac{a_2 b \operatorname{sign} y}{p}\tan^{-1}\frac{2\sqrt{m_1^2 + n^2 y^2} - a_2 bx \operatorname{sign} y}{2px} \\ &\left(m_1 = \frac{m}{H}, \quad b = n^2 \frac{I_1}{I_3}, \quad p^2 = b - \frac{a_2^2 b^2}{4}\right) \end{aligned}$$

Here D is an arbitrary constant.

Consider the crossing of the representative point of the coordinate axes. From the first equation of system (3.3) we have

$$x' = -\frac{H}{I_1} y - \frac{a_1 a_2}{I_1} \sqrt{m^2 + (I_2 x' + Hy)^2}\operatorname{sign} x$$

Substituting x' into the second equation we will obtain

$$I_3 y' - Hx = -a_2 \left[m^2 + \left(n^2 H^2 y^2 - a_1 a_2 \frac{I_2}{I_1} \sqrt{m^2 + (I_2 x' + Hy)^2}\operatorname{sign} x\right)^2\right]^{1/2} \operatorname{sign} y$$

Let the bearing friction on the axes of the suspension be so small that the terms multiplied by $a_1 a_2^2$ may be neglected. Then (3.3) becomes

$$I_3y' - Hx = -a_2\sqrt{m^2 + n^2H^2y^2}\,\mathrm{sign}\,y$$
$$I_1x' + Hy = -a_1a_2\sqrt{m^2 + n^2H^2y^2}\,\mathrm{sign}\,x \qquad (3.4)$$

It is easy to see that there is on the x-axis a slide region for the representative point defined by the inequality $|x| < a_2m/H$ tending towards the origin of the coordinates. Imagine that at the beginning of motion the representative point was located on the boundary of this region, i.e.

$$\alpha = \alpha_0, \qquad \beta = 0, \qquad \alpha' = x = a_2\frac{m}{H}, \qquad \beta' = 0 \qquad \text{for } t = 0.$$

The representative point will move further along the x-axis; therefore the inner ring remains immovable relative to the outer ring. The velocity of the latter is defined by the formula

$$\alpha' = a_2\frac{m}{H} - a_1a_2\frac{m}{I_1}t$$

Equating α' to zero, we find the time of motion τ_1 of the gyroscope

$$\tau_1 = \frac{I_1}{a_1H}$$

During this time the deviation along the outer ring axis attains the magnitude

$$\alpha = a_2\frac{m}{H}\tau_1 - \frac{1}{2}a_1a_2\frac{m}{I_1}\tau_1^2 = \frac{I_1}{2}\frac{a_2}{a_1}\frac{m}{H^2} = \frac{I_1}{4}\frac{a_2}{a_1}\frac{Pl_2}{H^2}$$

Example. Let there be given a gyroscope with the following parameters:

$I_1 = 11$ g cm sec^2, $P = 300$ g, $l = 5$ cm, $H = 7500$ g cm sec, $a_1 = a_2$

The deviation with respect to α for this gyroscope during the slide time of the representative point along the x-axis attains $0.25'$.

The slide region on the y-axis is determined by the inequality

$$|y| \leqslant a_1a_2\frac{m}{H}$$

The deviation of the gyroscope with respect to the angle β during the sliding of the representative point on the y-axis is quite negligible.

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